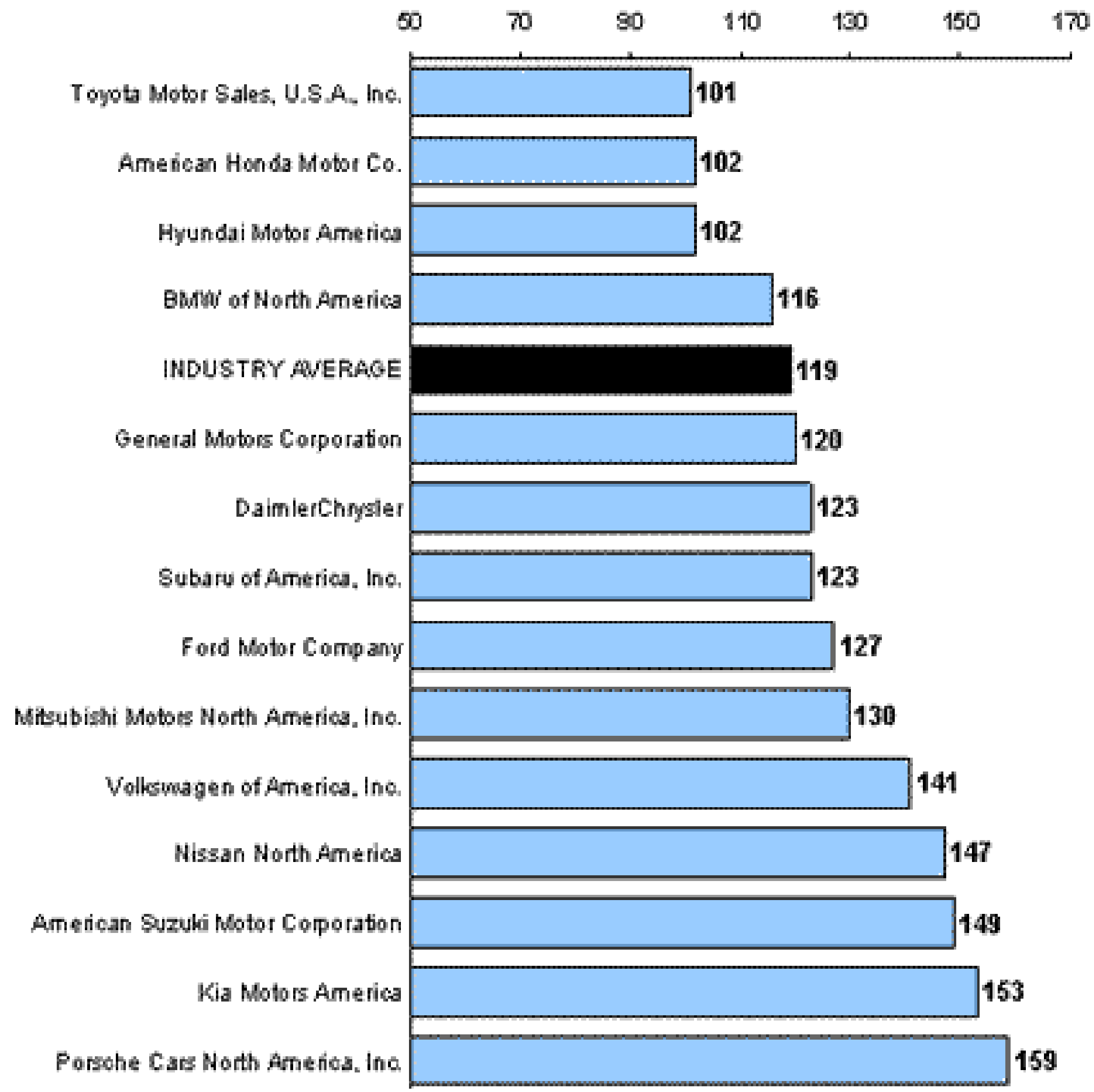
VARIATION AND QUALITY CONTROL IN MANUFACTURING

Sanjay Sarma

J.D. Power and Associates
2004 Initial Quality Study℠ (IQS)

2004 Corporate IQS Ranking

Problems Per 100 Vehicles



The root of all evil in manufacturing

is variation. **Variation** is inevitable.

Causes of Variation:

- Random
- Assignable

Types of Variation:

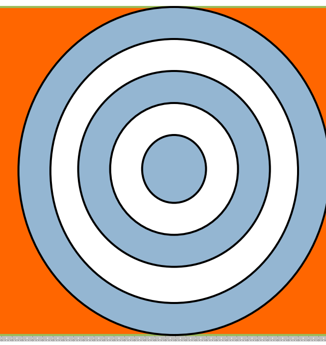
- Physical var. → Quality
- Temporal var. → Scheduling/ Systems

Our schedule

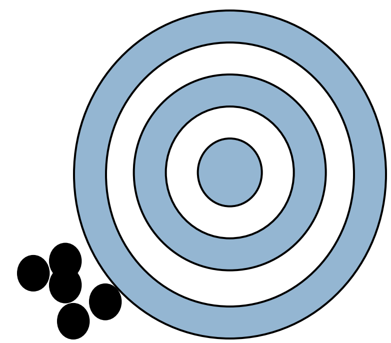
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Notes:						1
2	3 Assembly II	4	5 Quality Control I	6	7	8
9	10 Veteren's Day No Classes	11 No Classes	12 Quality Control II	13	14	15
16	17 Manufacturing Systems I	18	19 Manufacturing Systems II	20	21	22
23	24 Manufacturing Systems III	25	26 Manufacturing Cost	27 Thanksgiving Day	28	29
30	Notes:					

3 Things Matter in Quality

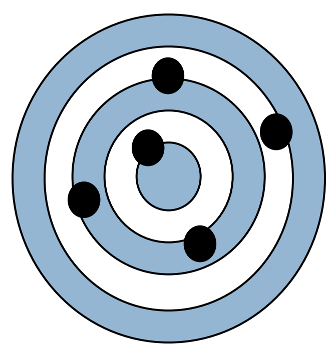
- The SIZE of the target



- Repeatability

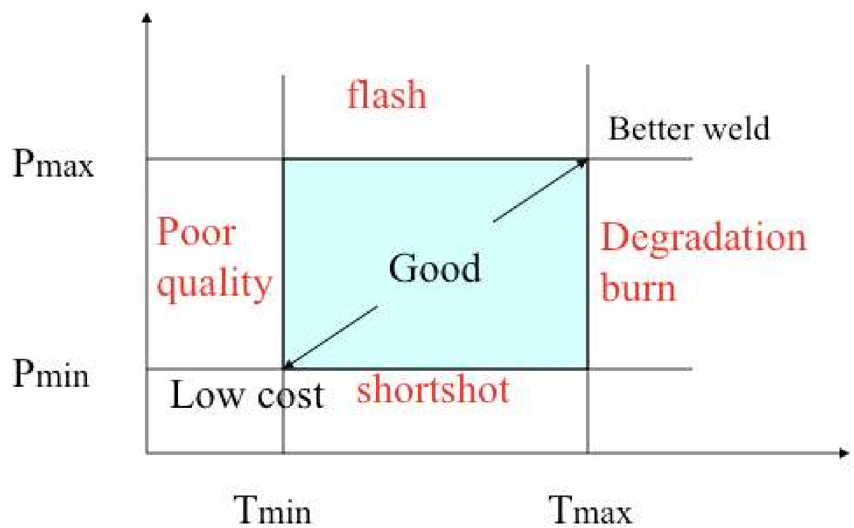


- Accuracy



The Size of the target

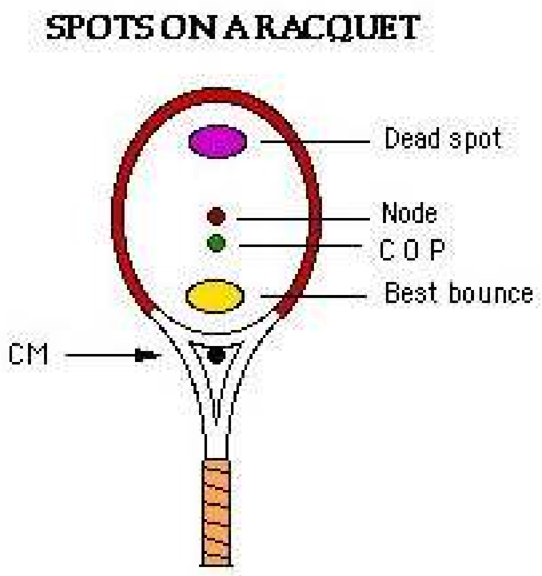
- The target is the process window

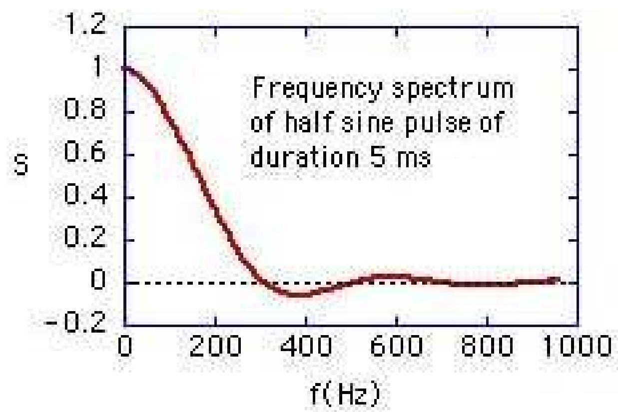


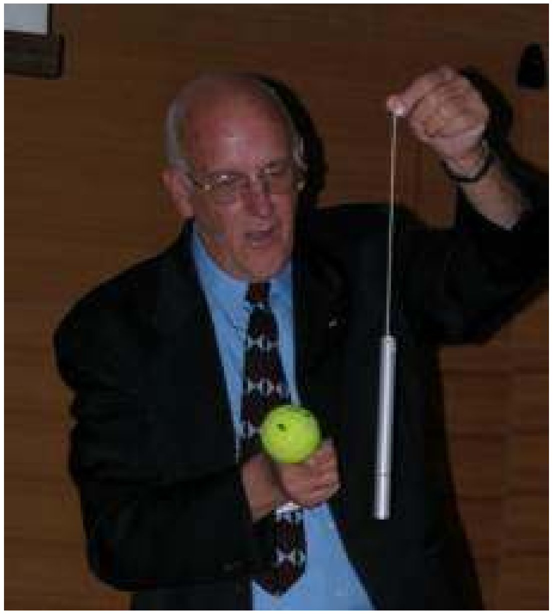
- Manufacturing process design is all about making the window big!
- This is called robust design.

Big targets are hard to miss

- Consider the sweet spot of a tennis racket



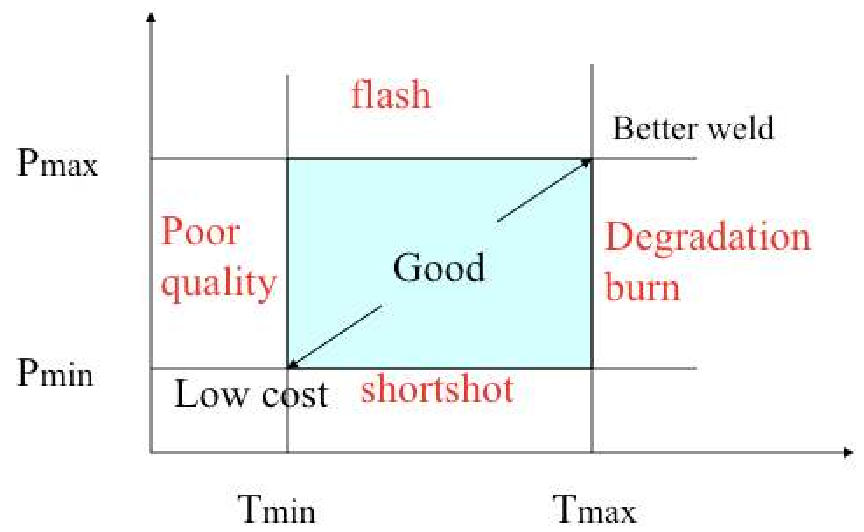




http://www.physics.usyd.edu.au/~cross/tennis.html

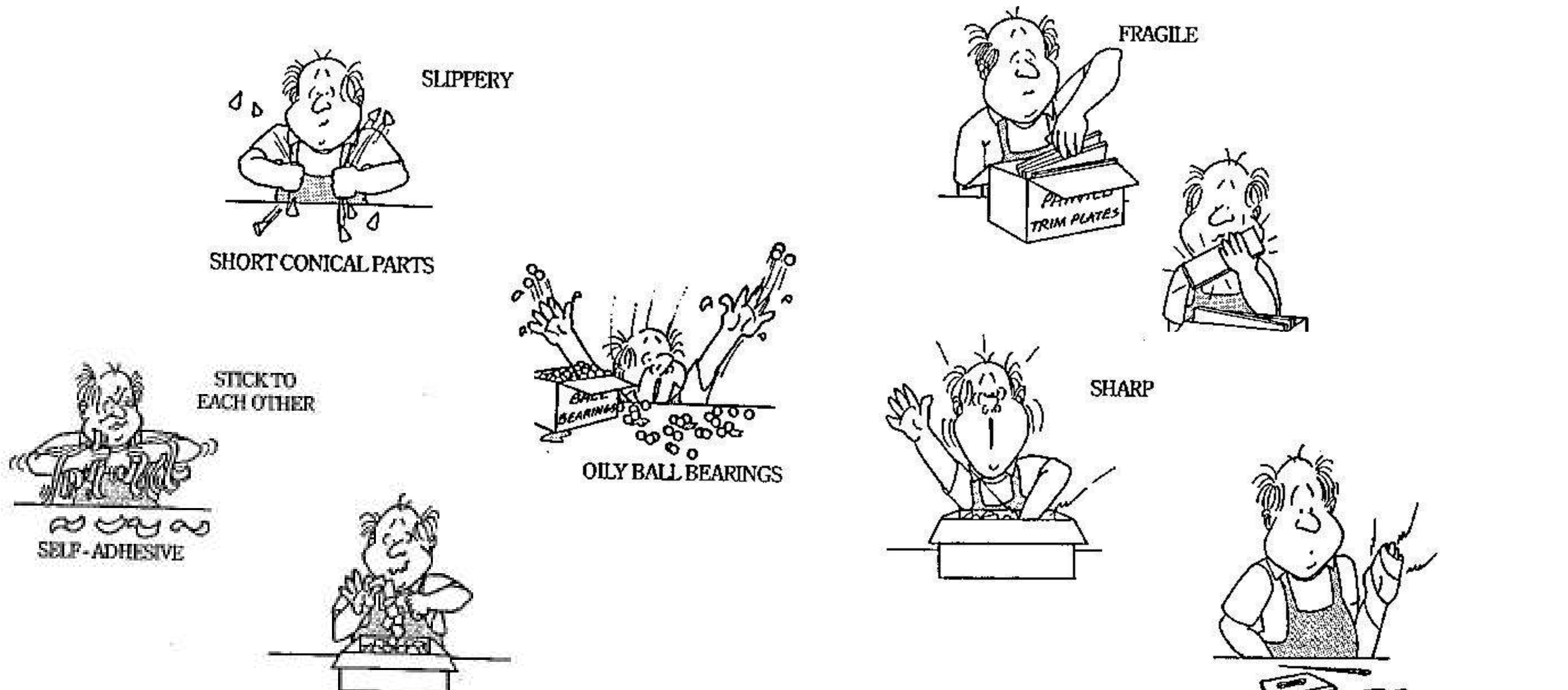
How to make the injection molding robust

1. Formulation of plastic:
   - Low viscosity
   - High burn temperature
   - Medium melting point
2. Passive locks to help hold molds together
3. Better mold design



How to make assembly robust

- “Should be easy for a blind person wearing boxing gloves.”



Robust process design is key

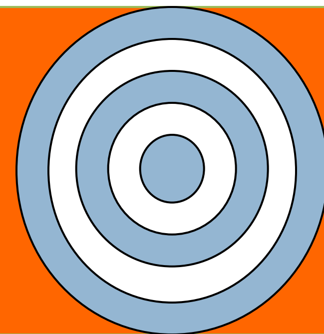
To a number of consumer products

- Cake mixes
  - Used to be unforgiving…
  - Only now are they forgiving enough for home use
- Do-it-yourself processes
  - Home bread, beer making
- Adhesives for home use
- Printers

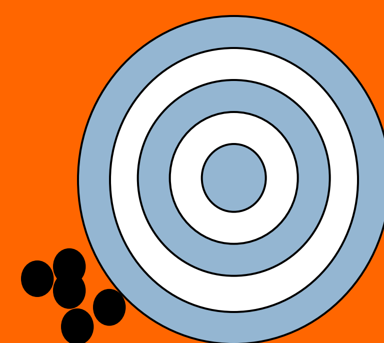
KEY IS MAKING PROCESSES **FORGIVING**

3 Things Matter in Quality

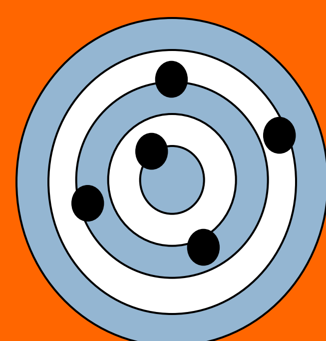
- The SIZE of the target



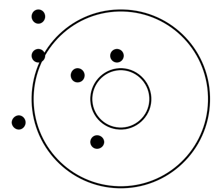
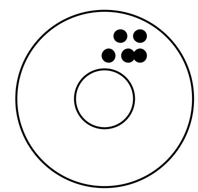
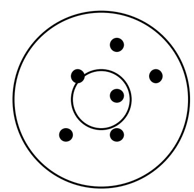
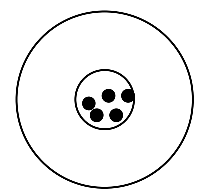
- Repeatability



- Accuracy



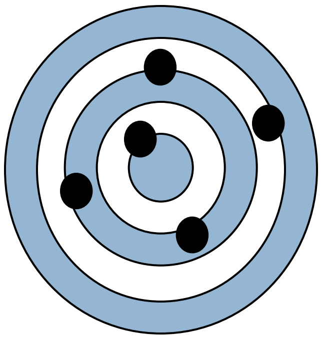
Variations: Reminder

	Not precise	Precise
Not accurate		
Accurate		

Thought experiment(s)

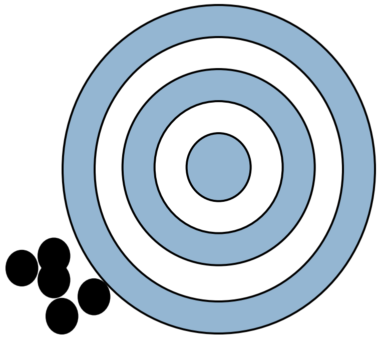
Say you are injection-molding yo's and measuring diameters.

1. What is a good measure of the accuracy of a process?



$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

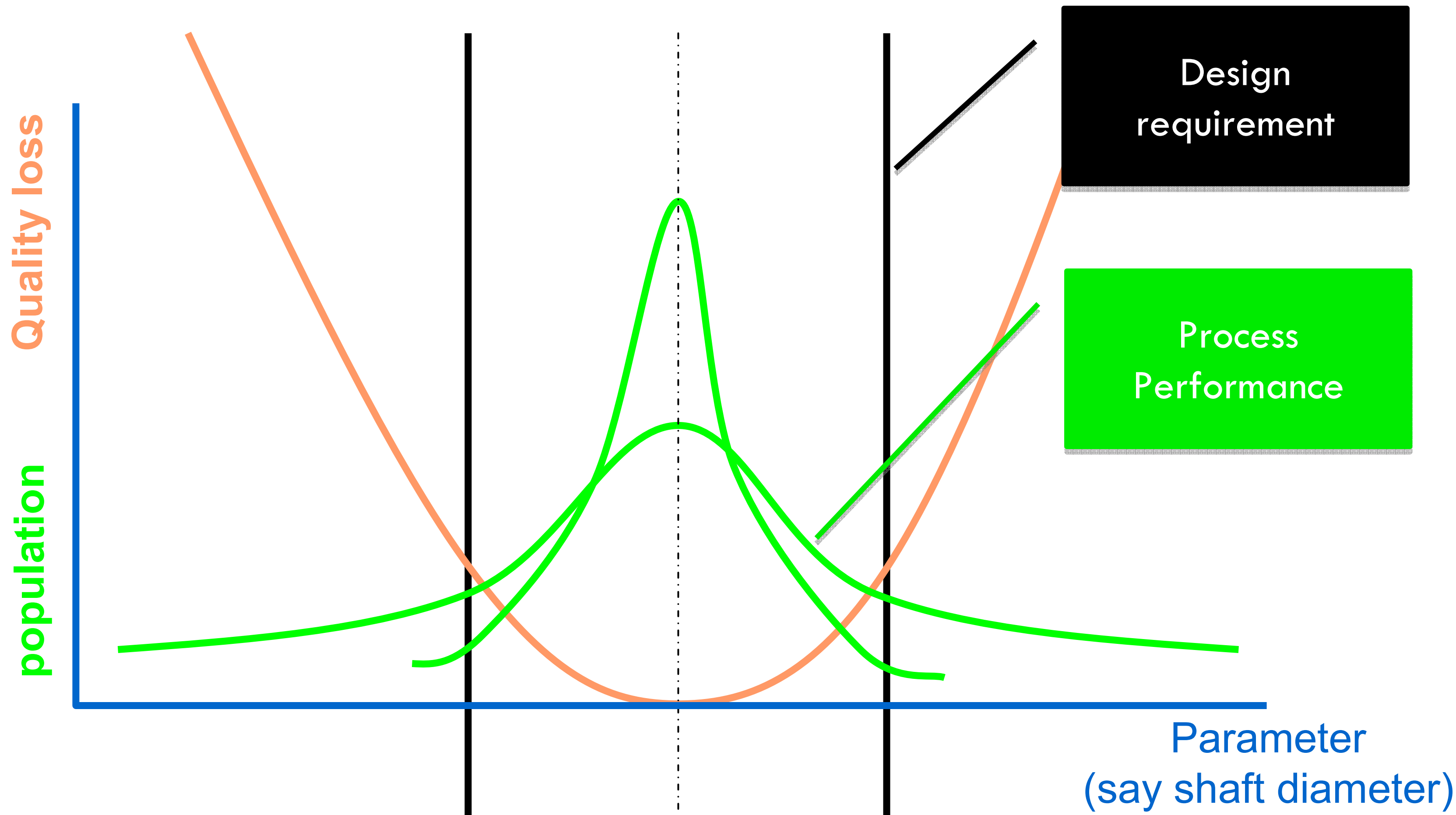
2. What is a good measure of the repeatability of a process? Same "yo's" as above.

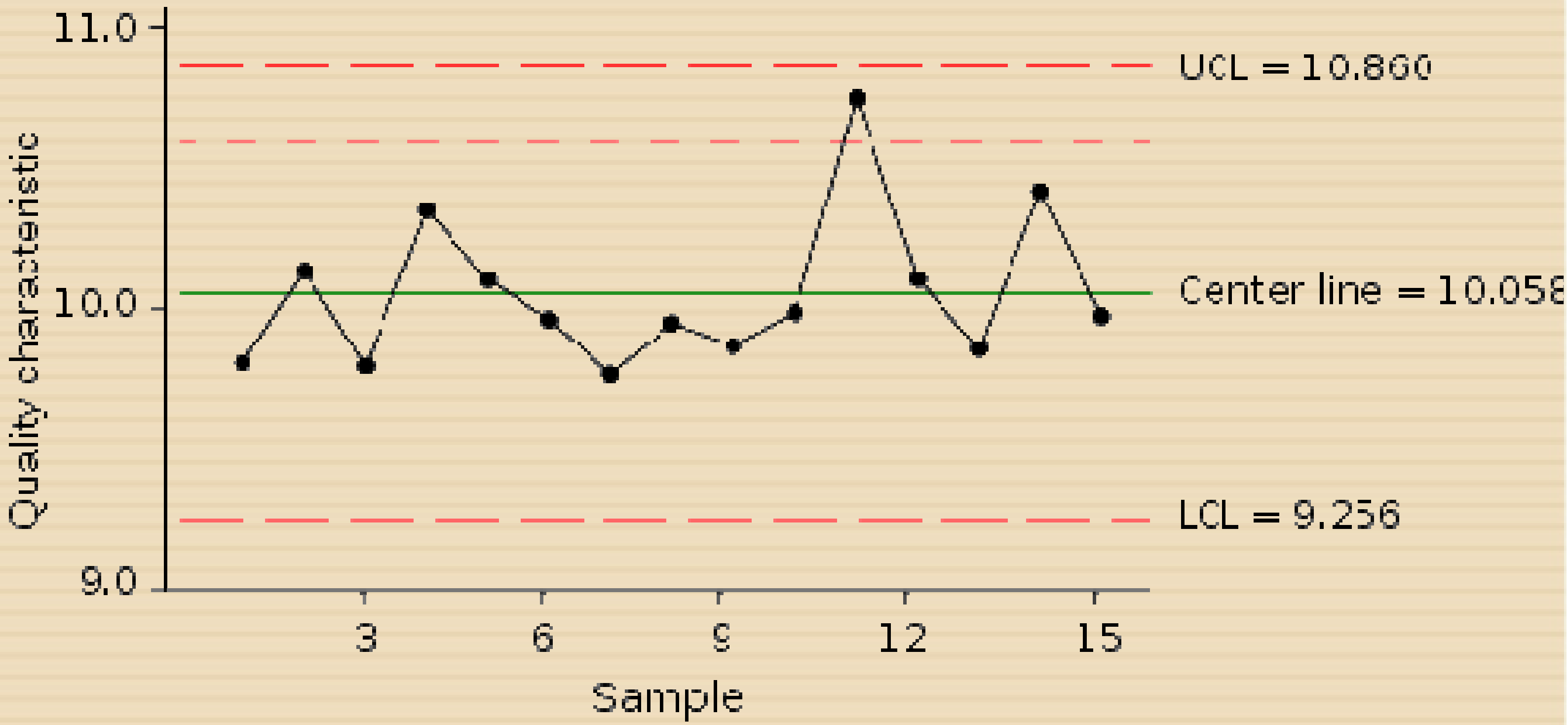


$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n}}$$

Why does variation hurt, exactly?

- Stuff doesn't fit together very well





TRACKING PROCESS PERFORMANCE

This is called **Process Control**

Statistics basics

- Central tendency
  - Sample mean (arithmetic):
  - Sample median

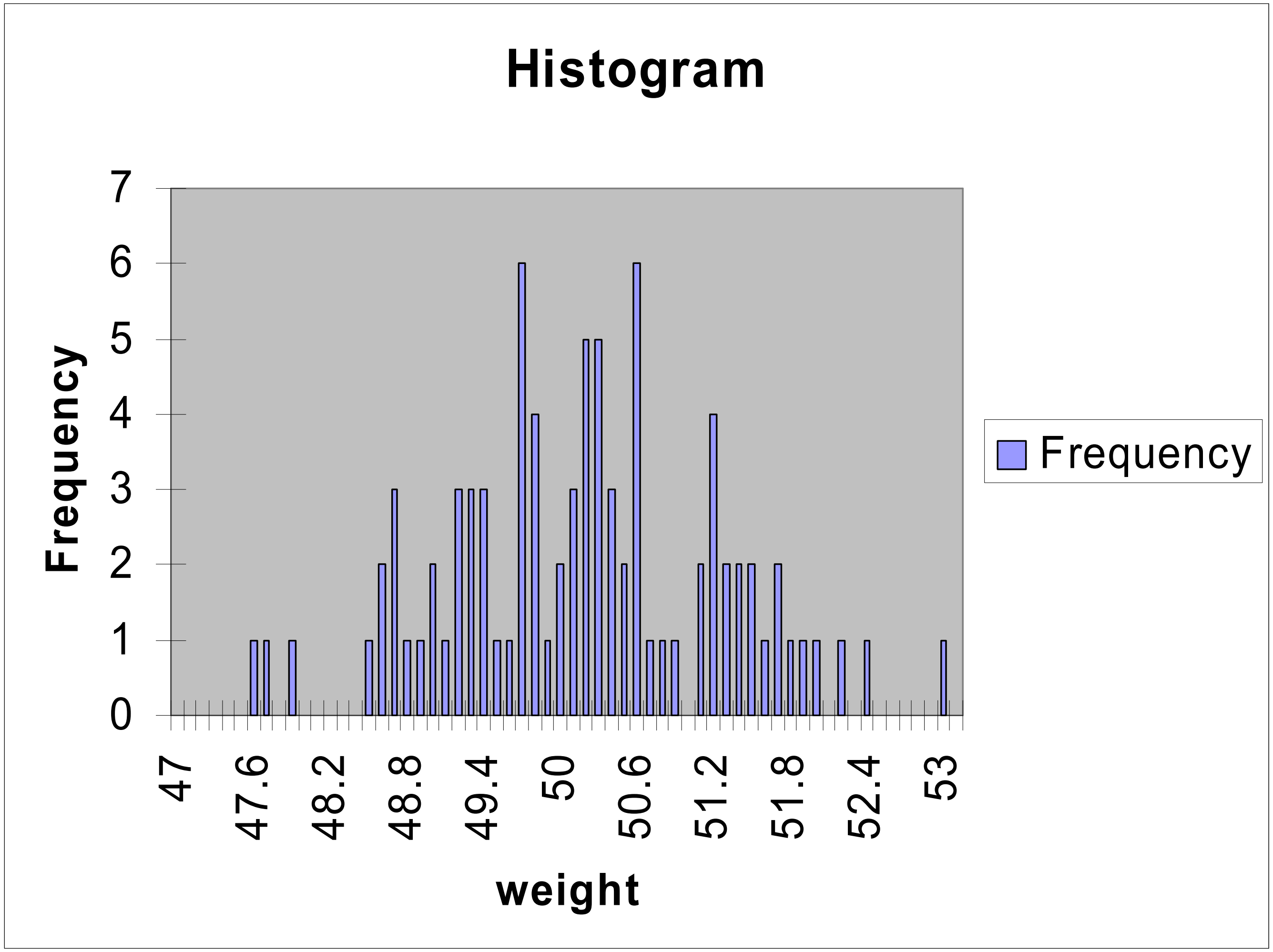
$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

- Measures of dispersion
  - Standard deviation
  - Variance
  - Range

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}}$$

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$$

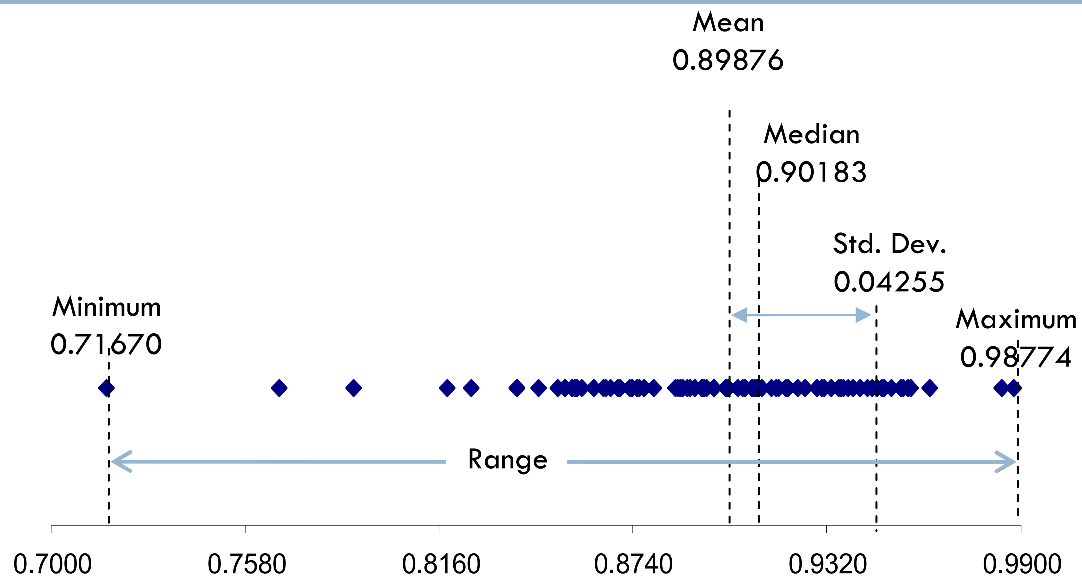
m&m net weight distribution



n=88
Mean:50.17
S=1.08

Package weight
0.9 g

Measures of M&Ms' dispersion



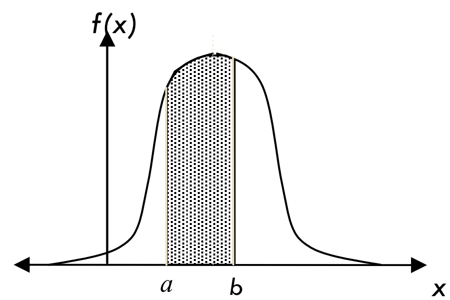
- Standard deviation, Range measure spread
- Mean measures, well, mean

Normal Probability Density Function

$$f(x) = \frac{1}{\sqrt{2\pi}s} \cdot e^{\left(-\frac{(x-\bar{x})^2}{2s^2}\right)}$$

Probability

$$P\{a \le x \le b\} = \int_a^b f(x)dx$$

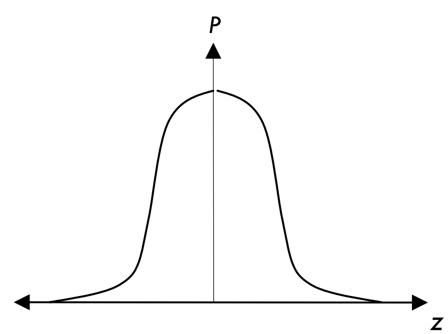


$$P\{-\infty \le x \le \infty\} = \int_{-\infty}^{\infty} f(x)dx = 1$$ For all $\bar{x}$, s

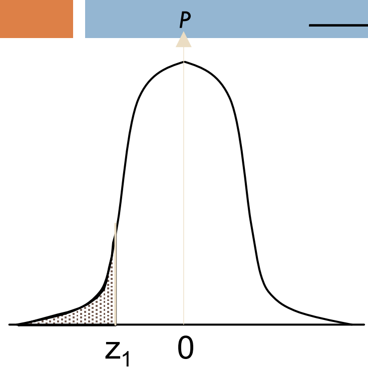
Normalized

$$z = \frac{x - \bar{x}}{s}$$

$$P\{z_1 \le z \le z_2\} = \int_{z_1}^{z_2} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$



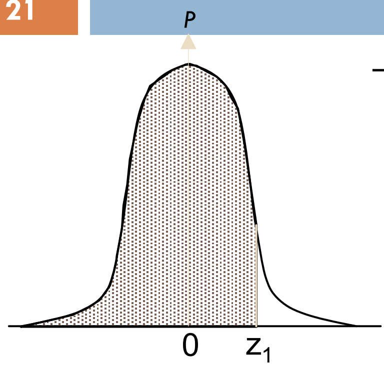
Areas under the Normal Distribution Curve



$$\int_{-\infty}^{-0.55} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
replace	0.50000	0.49601	0.49202	0.48803	0.48404	0.48006	0.47607	0.47209	0.46811	0.46414

Areas under the Normal Distribution Curve



Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Normal Distribution Example

Take an M&M with mass = 0.9g, based on our calculated normal curve, how many M&M's have a mass greater than 0.9g?

Mean
0.89876

Std. Dev.
0.04255

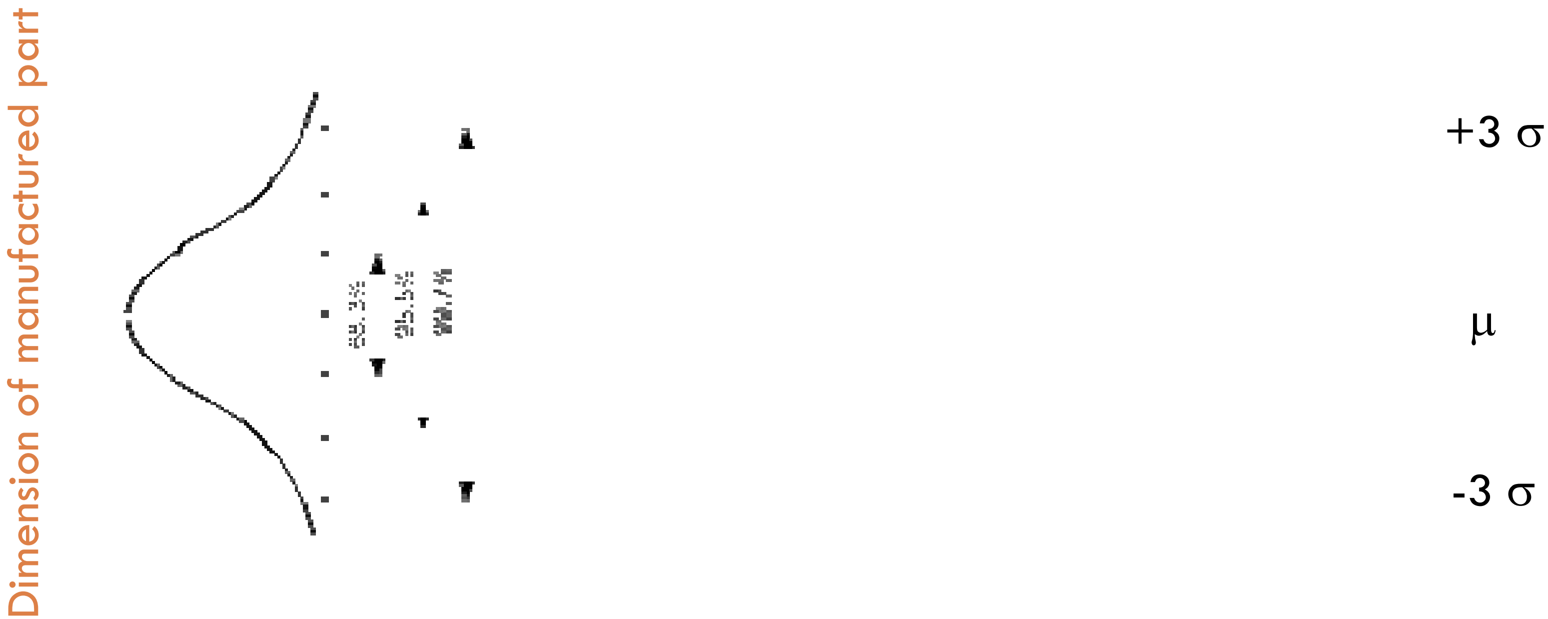
$$Z = \frac{x - \overline{x}}{s}$$

Z = (0.9 - 0.89876) / 0.04255 = 0.29

Area to the right of Z=0.29, from table on previous page:
P = (1 - 0.6141) = 0.3859

So, the number of M&M's with a mass greater than 0.9g
= P*n = 0.3859 * 100 = 39

Rudimentary Run Charts: The EKG of processes

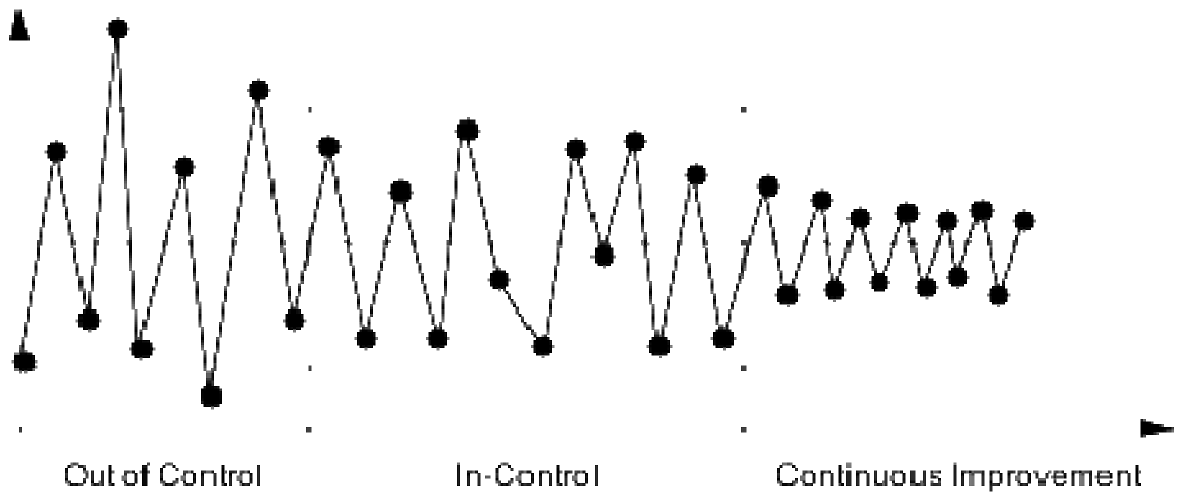


By reading the Shewhart Chart

It is possible to see…

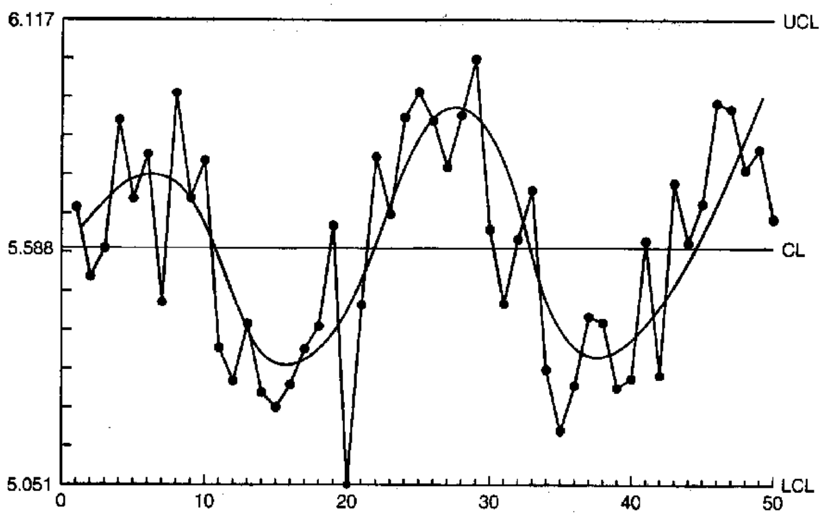
- if the process is running steady
- If the tool is wearing out
- If some new vibration has occurred
- If a setting is off
- If a new batch of raw materials has different properties
- If an operator is sleepy

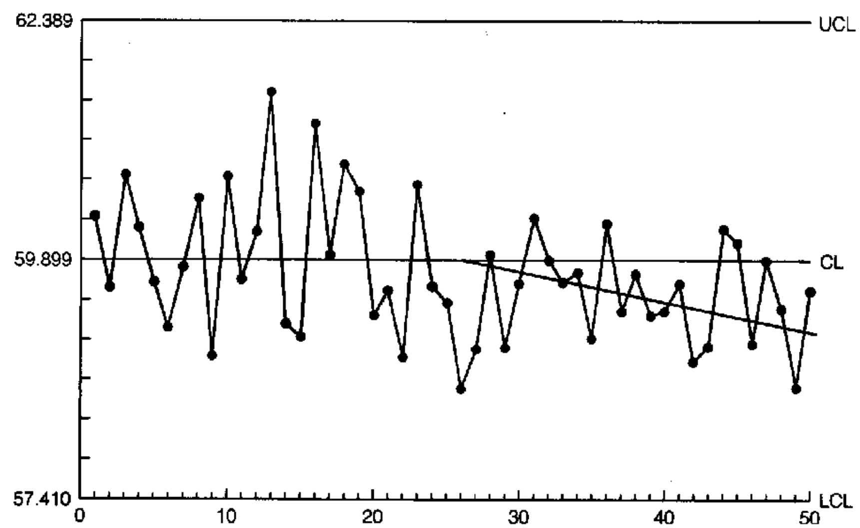
Continuous improvement



The Japanese call is “Kaizen”

Patterns


6.117
UCL
5.588
CL
5.051
LCL
0
10
20
30
40
50


62.389
UCL
59.899
CL
57.410
LCL
0
10
20
30
40
50

Patterns continued

Statistically In-Control:

A process under surveillance (monitoring) by periodic sampling maintains constant mean level and level of variability over time.

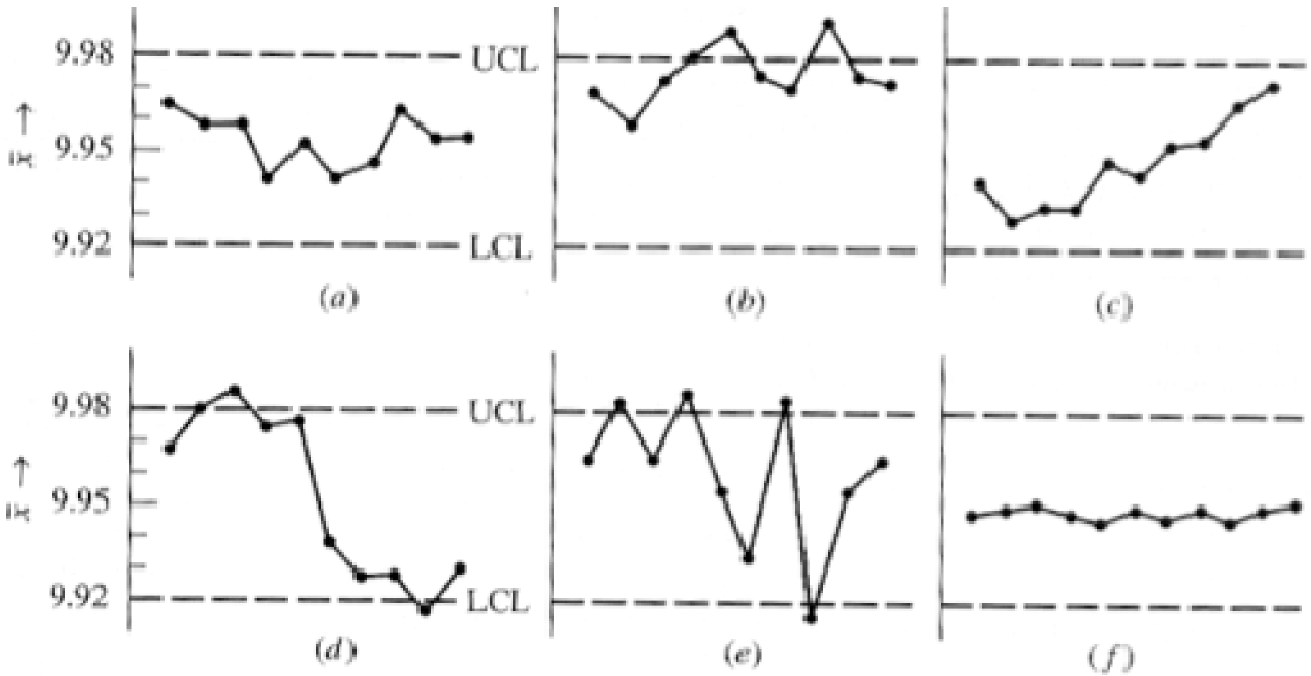
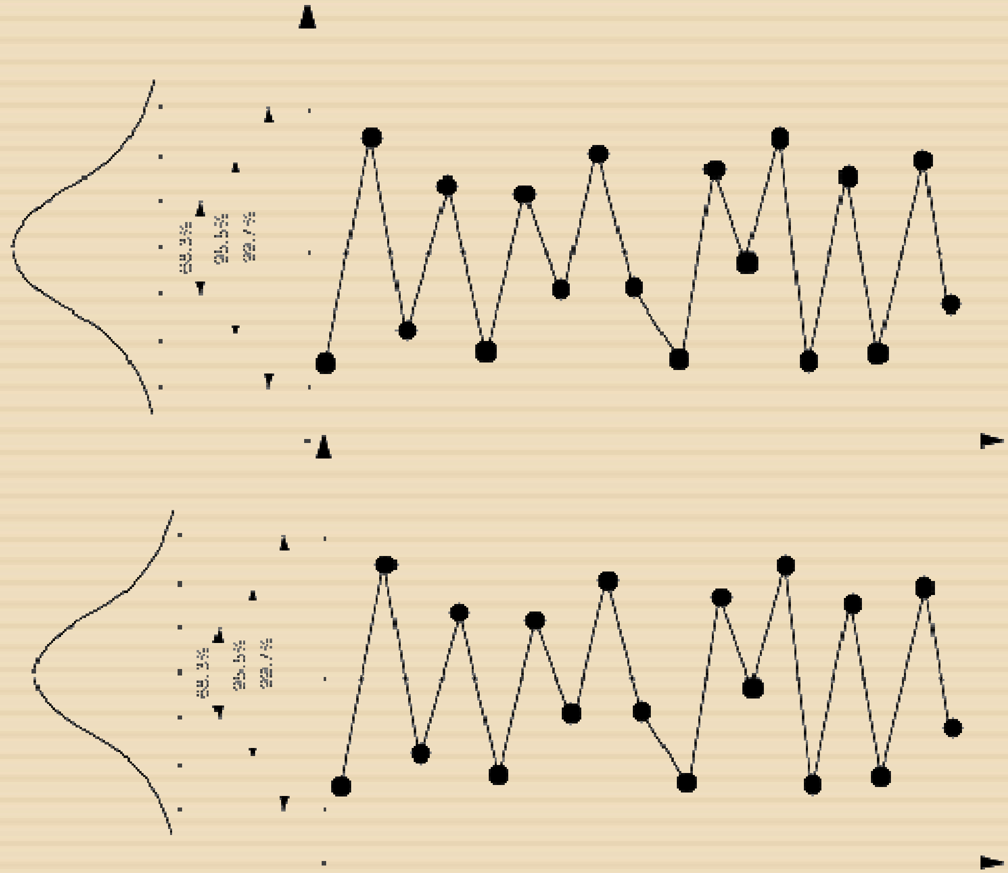


Figure 21–10 Statistical process control gives valuable clues: (*a*) process is in statistical control, (*b*) improperly set tooling, (*c*) rapid tool wear, (*d*) mixed material lot of two hardnesses, (*e*) process out of control, and (*f*) something is going very well and is worth investigating.



Advanced Control Charts

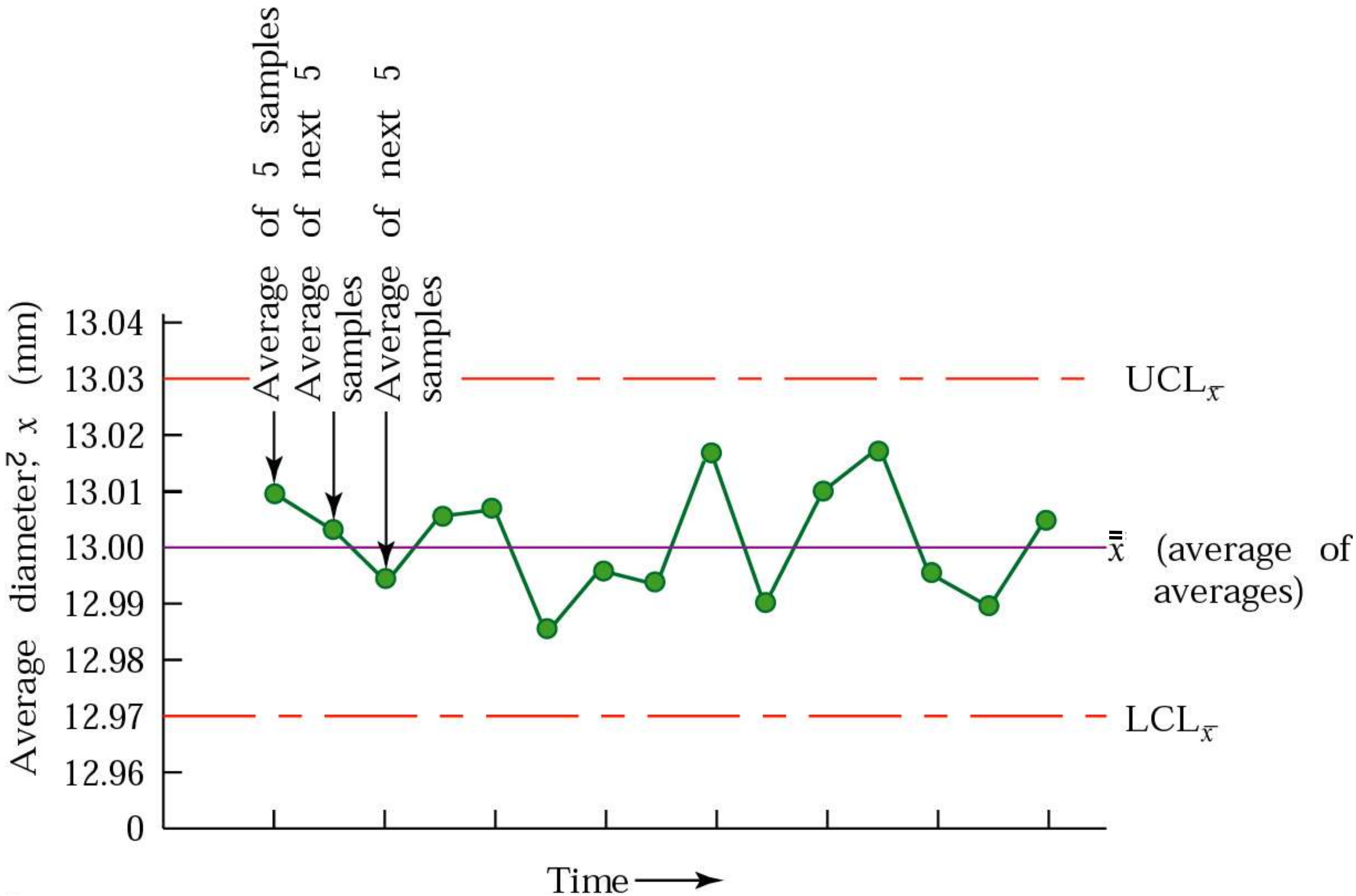
Now we see how they really do it

2 new concepts (extensions)

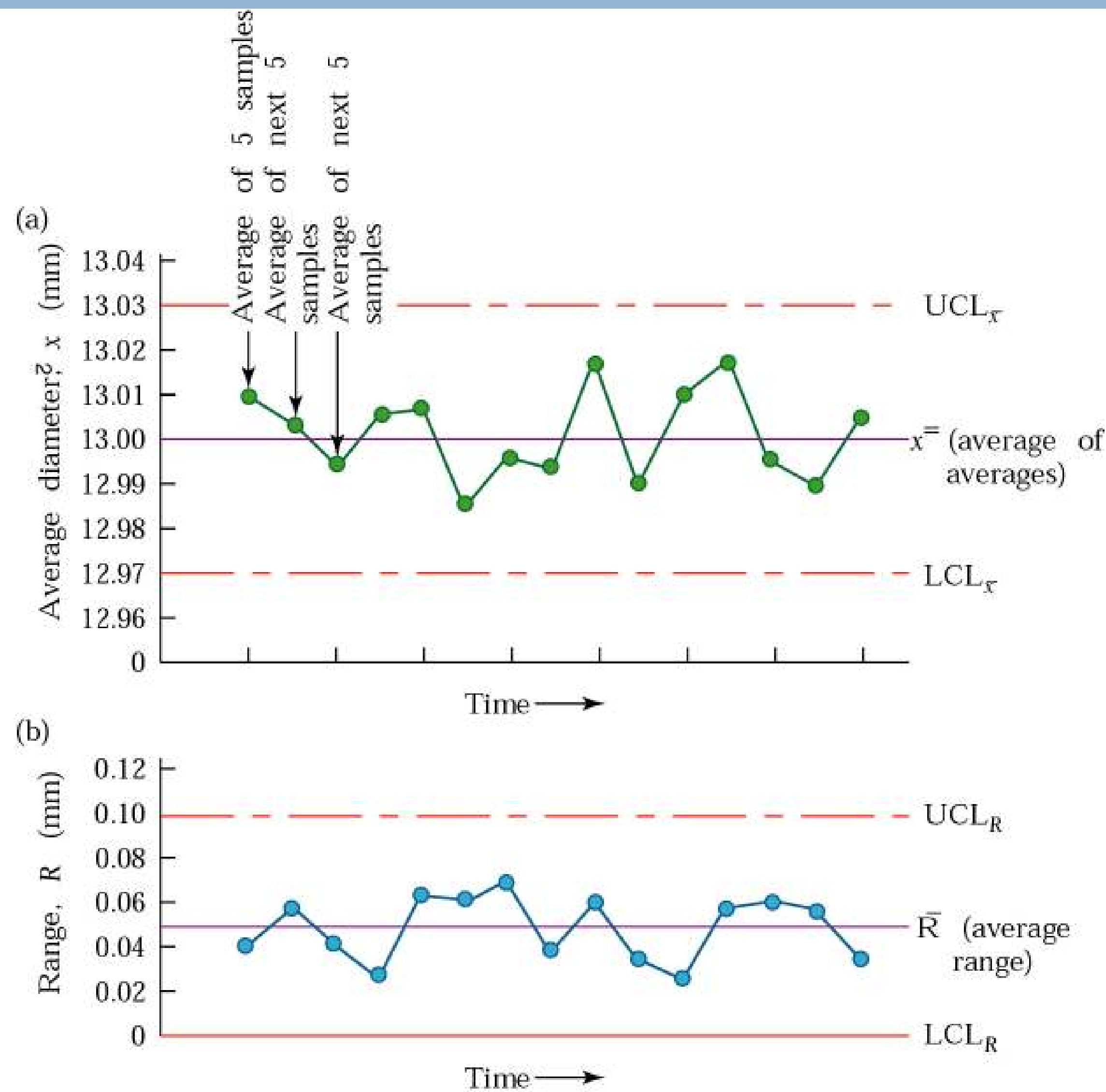
1. Sub-group sampling
   - Hard to measure every part
   - Sub-groups can be almost as telling
2. R-Charts
   - Sometimes it is easier to look at range of subgroup
   - Can get similar results

All this statistically sound.

Sub-Groups


Average of 5 samples
Average of next 5 samples
Average of next 5 samples
Average diameter, x (mm)
13.04
13.03
13.02
13.01
13.00
12.99
12.98
12.97
12.96
0
UCL
LCL
(average of averages)
Time

Statistical Quality Control



Procedure for computing charts

1. **For each sample, an average is calculated**

$$\bar{X}_i = \frac{\sum_{j=1}^{n} X_{ij}}{n} \quad \text{where } X_{ij} \text{ is the j-th measurement in the i-th sample.}$$

2. **Calculate the range within the sample:**

$$R_i = X_{largest} - X_{smallest}$$

3. **Calculate the GRAND AVERAGE:**

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{N} \bar{X}_i}{N}, \text{ where N is the number of subgroups.}$$

Procedure continued

4. **Calculate the average of the sample RANGES:**

$$\overline{R} = \frac{\sum_{i=1}^{N} R_i}{N}$$

5. **Control limits for R-chart**

$$LCL = D_3\overline{R}, \quad UCL = D_4\overline{R}$$

6. **Control limits for X-bar chart**

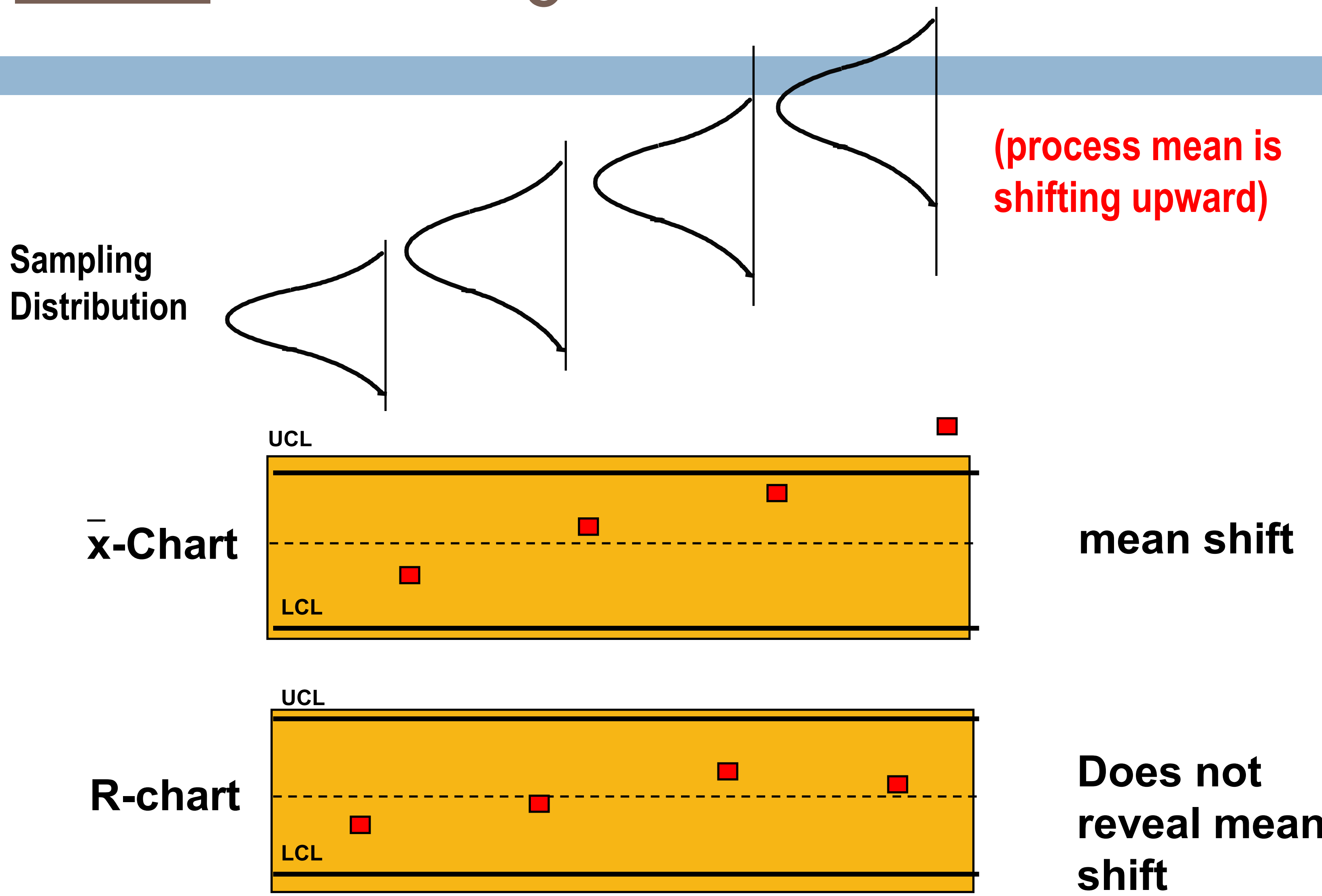
$$UCL = \overline{\overline{X}} + A_2\overline{R}$$
$$LCL = \overline{\overline{X}} - A_2\overline{R}$$

where A_2 D_3, and D_4 are functions of sample size.

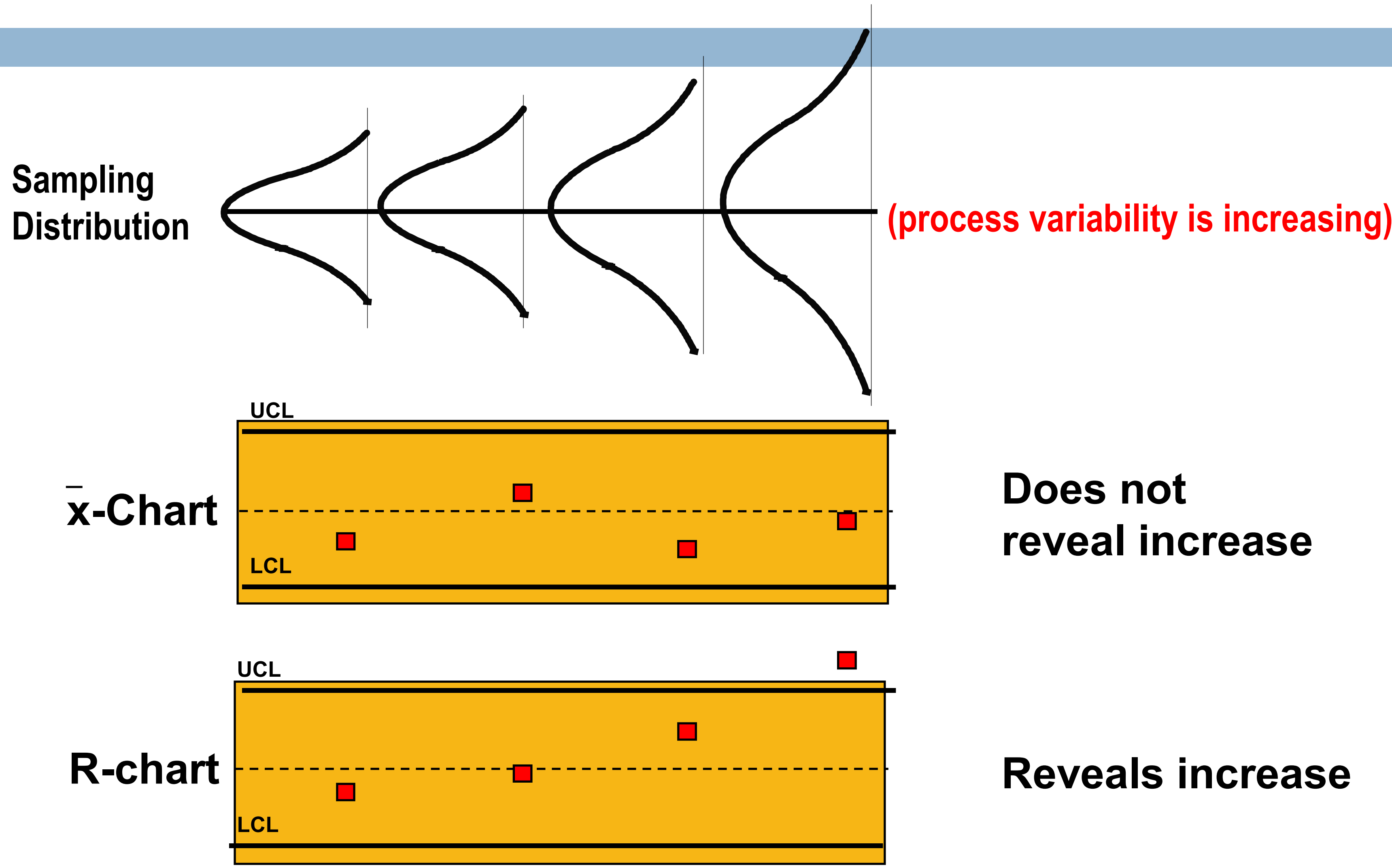
Factors for computing control chart limits

Sample size	c_4	d_2	d_3	D_3	D_4	A_2
2	0.7979	1.128	0.853	0	3.267	1.880
3	0.8862	1.693	0.888	0	2.574	1.023
4	0.9213	2.059	0.880	0	2.282	0.729
5	0.9400	2.326	0.864	0	2.114	0.577
6	0.9515	2.534	0.848	0	2.004	0.483
7	0.9594	2.704	0.833	0.076	1.924	0.419
8	0.9650	2.847	0.820	0.136	1.864	0.373
9	0.9693	2.970	0.808	0.184	1.816	0.337
10	0.9727	3.078	0.797	0.223	1.777	0.308
11	0.9754	3.173	0.787	0.256	1.744	0.285
12	0.9776	3.258	0.778	0.283	1.717	0.266
13	0.9794	3.336	0.770	0.307	1.693	0.249
14	0.9810	3.407	0.763	0.328	1.672	0.235
15	0.9823	3.472	0.756	0.347	1.653	0.223
16	0.9835	3.532	0.750	0.363	1.637	0.212
17	0.9845	3.588	0.744	0.378	1.622	0.203
18	0.9854	3.640	0.739	0.391	1.608	0.194
19	0.9862	3.689	0.734	0.403	1.597	0.187
20	0.9869	3.735	0.729	0.415	1.585	0.180

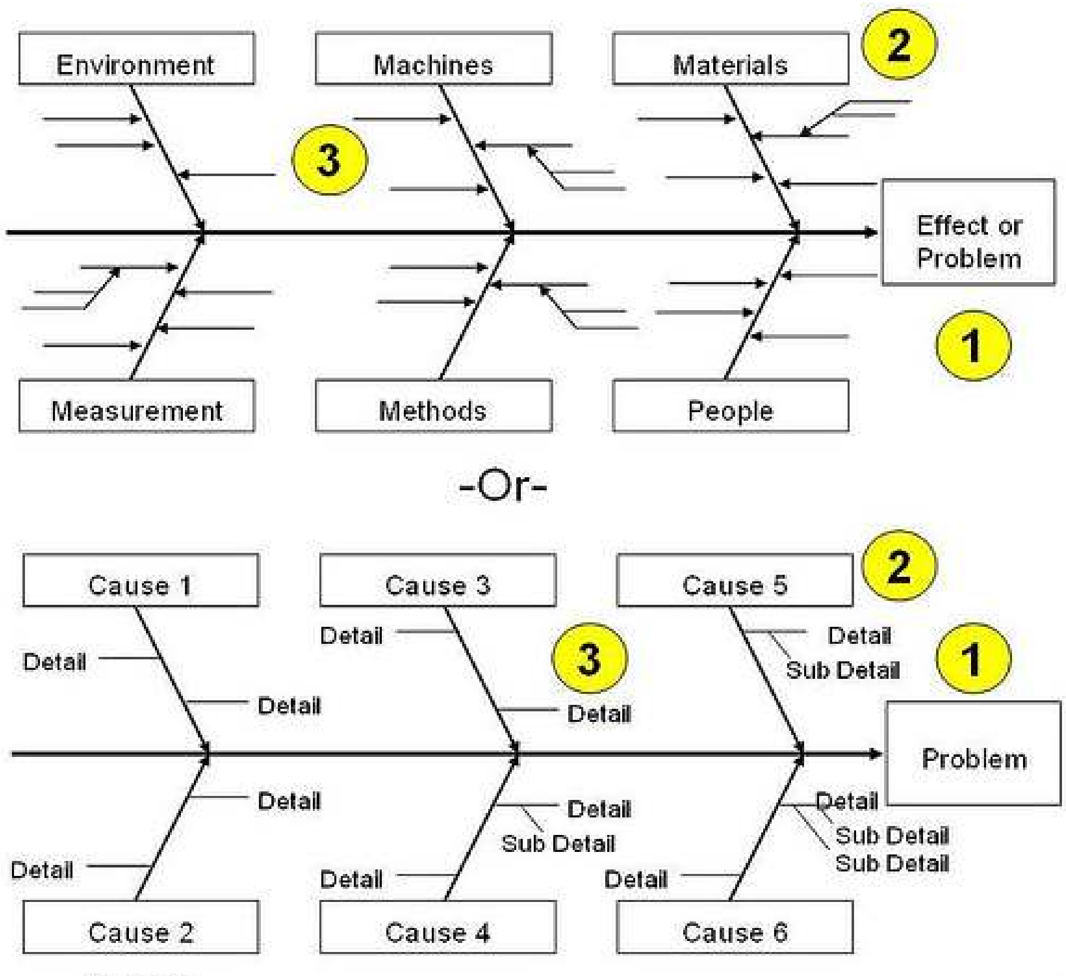
Mean and Range Charts



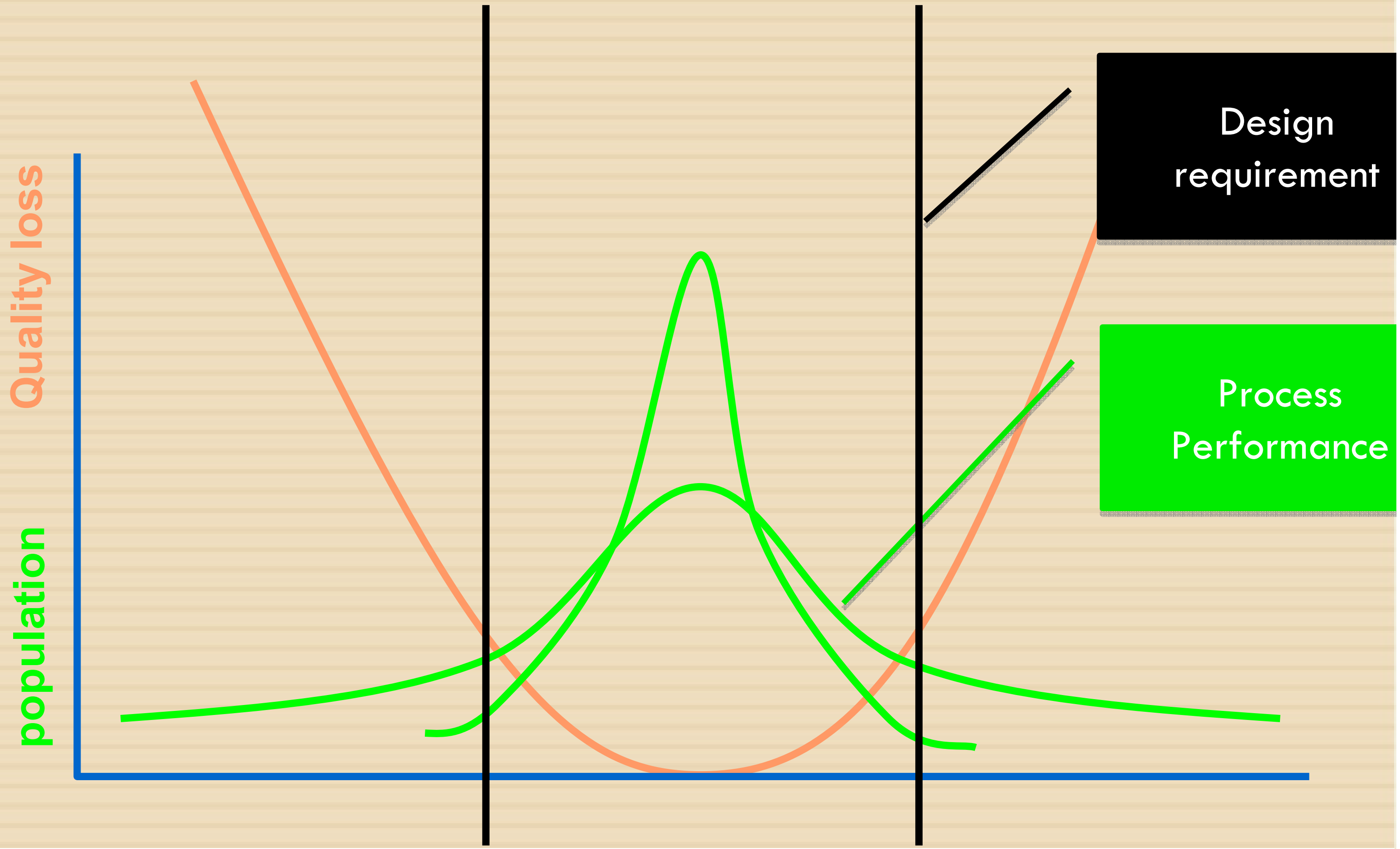
Mean and Range Charts



Using the EKG to debug the patient



Ishikawa fishbone diagram



OK, now we understand the process

Let’s understand how it compares to what we need

Tolerance versus Control

Control Limits are:

- • Based on process mean and variability
- • Dependent on the sampling parameters,
- • Used to identify special causes,
- • Characteristic of the process in question.

Specification Limits or Tolerance are

- • Based on functional consideration,
- • Related to individual part measurement,
- • Used to establish a part's conformability to design,
- • Characteristic of the part in question.

Process Capability Index C_p

Comparison of process variation with the specifications.

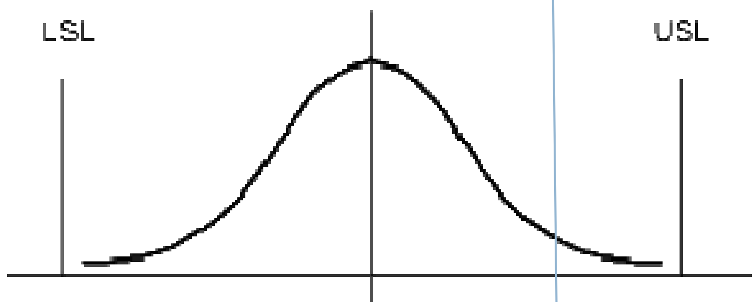
$$C_p = \frac{USL - LSL}{6\,\sigma_X}$$

Cp should be 1.33 or larger.

Upper specification limit

Lower specification limit

Then we say that the process is "capable."



Narrative

- This shop has been producing shafts for the last few months.
- The σ for this shop has been 0.01mm
- The design requirement is that the shaft be: 10mm $\pm$0.04mm
- The tolerance band: USL - LSL is 0.008m
- Cp =1.333

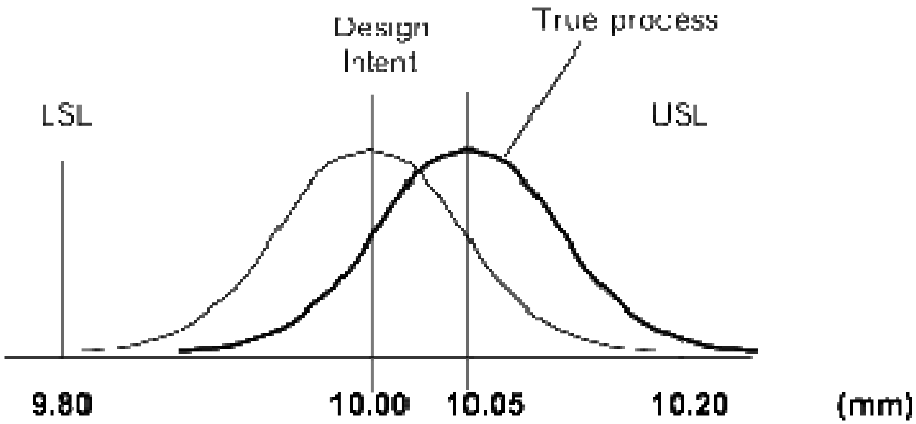
According to our company standard,

the process is capable

C_p does not capture mean shift

Comparison of process variation and mean with the specifications.

$$C_{pk} = \frac{USL - \mu_x}{3\sigma_x} \text{ or } \frac{\mu_x - LSL}{3\sigma_x} \text{ whichever is smaller.}$$



Quality is everything

- Intel, for *e.g.*, is a manufacturing company
  - Their competitive advantage is quality
  - They can make smaller, faster chips cheaper than other companies.

<u>The Secret Summary</u>

- Enlarge process windows (Process, part design)
- Keep your process in control (Control charts)
- Ensure that the process suits your design ($C_{p,}$ C_{pk})

External credits

- MIT slides past and present
- Scans from various books
- courses.**washington.edu/courseli/me355/Lectures/Sp2006/Lecture%2022%20 -%20SPC,%20CP.ppt**